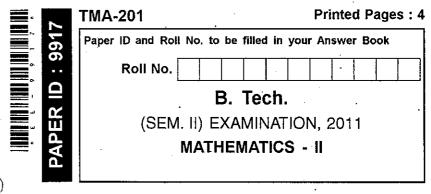
- (v) Find the particular integral of $(D-D'-1)(D-D'-2)z = \sin(2x+3y)$ where $D \equiv \frac{\partial}{\partial x}$, $D' \equiv \frac{\partial}{\partial y}$.
- (vi) Show that the equation $\frac{\partial^2 z}{\partial x^2} + 2x \frac{\partial^2 z}{\partial x \partial y} + \left(1 y^2\right) \frac{\partial^2 z}{\partial y^2} = 0$ is elliptic for all values of x and y in the region $x^2 + y^2 < 1$, parabolic on the boundary and hyperbolic out side the region.
- 5 Attempt any two of the following: 10×2=20
 - (i) A tightly stretched string with fixed end points x = 0 and x = l is initially in a position given by $y(x, 0) = y_0 \sin(\pi x/l)$. If it is released from rest from this position, find the displacement y at any distance x from one end at any time t.
 - (ii) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, $0 < x < \pi$, $0 < y < \pi$, which satisfies the conditions: $u(0, y) = u(\pi, y) = u(x, \pi) = 0$ and $u(x, 0) = \sin^2 x$.
 - (iii) A rod of length L has its ends A and B kept at $0^{\circ}C$ and $100^{\circ}C$, respectively, until steady state conditions prevail. If the temperature of B is then reduced suddenly to $0^{\circ}C$ and kept so, while that of A is maintained, find the temperature u(x, t) at distance x from A at time t.



Time: 3 Hours]

[Total Marks: 100

Attempt all questions.

1 Attempt any four of following:

 $5 \times 4 = 20$

(i) Solve:
$$x dx + y dy = \frac{a^2 (x dy - y dx)}{x^2 + y^2}$$
.

(ii) Solve:
$$\frac{dz}{dx} + \frac{z}{x} \log z = \frac{z}{x^2} (\log z)^2$$

(iii) Solve:
$$(D^2 + a^2)y = \sec ax$$
.

(iv) Solve the following simultaneous equations

$$\frac{dx}{dt} + \frac{dy}{dt} - 2y = 2\cos t - 7\sin t$$

$$\frac{dx}{dt} - \frac{dy}{dt} + 2x = 4\cos t - 3\sin t$$

(v) Solve:
$$\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} + \frac{a^2}{x^4}y = 0$$
.

(vi) Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2} + y = \tan x$.

- 2 Attempt any two of the following: 10×2=20
 - (i) (a) If f(t) is continuous and of exponential order a as $t \to \infty$, then prove that Laplace transform of f(t) exists for s > a.
 - (b) Find the Laplace transform of the following periodic function:

$$f(t) = \frac{t}{T}$$
 for $0 < t < T$.

(ii) If $\overline{f_1}(s)$ and $\overline{f_2}(s)$ are the Laplace transforms of $f_1(t)$ and $f_2(t)$ respectively, then prove that $\overline{f_1}(s) \cdot \overline{f_2}(s) = L \left\{ \int_0^t f_1(u) \cdot f_2(t-u) du \right\}.$

Hence evaluate
$$L^{-1}\left\{\frac{s^2}{\left(s^2+a^2\right)\left(s^2+b^2\right)}\right\}$$
.

(iii) Solve the following simultaneous differential equations by using Laplace transformation:

$$Dx - y = e^t$$
, $Dy + x = \sin t$, $x_0 = 1$, $y_0 = 0$.

- 3 Attempt any two of the following: 10×2=20
 - (i) Test the convergence or divergence of the following series:
 - (a) $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p} (p > 0)$

(b)
$$\sum_{n=1}^{\infty} \frac{1}{[n+\sqrt{(n+1)}]}$$

(ii) State and prove the Cauchy's root test. Hence test for convergence the following series

$$\sum_{n=1}^{\infty} \left(1 + \frac{1}{\sqrt{n}} \right)^{-n^{\frac{3}{2}}}$$

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(iii) (a) Show that the following series converges uniformly in any interval:

$$\sum_{n=1}^{\infty} \frac{\cos nx}{n^p}$$

(b) Prove that

$$\int_0^1 \left(\sum_{n=1}^{\infty} \frac{x^n}{n^2} \right) dx = \sum \frac{1}{n^2 (n+1)}$$

- 4 Attempt any four of the following: 5×4=20
 - Find the Fourier series for the function $f(x) = x + x^2$ in the interval $-\pi < x < \pi$. Hence show that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots$
 - (ii) Obtain Fourier series for the function f(x) given by $f(x) = 1 + 2x/\pi$, $-\pi \le x \le 0$ $= 1 - 2x/\pi$, $0 \le x \le \pi$
 - (iii) Express f(x) = x as a half-range sine series in 0 < x < 2
 - (iv) Eliminate the arbitrary function f and g from the following equation:

$$v = f(x+iy) + g(x-iy)$$
, where $i = \sqrt{(-1)}$