(v) Find the particular integral of

$$(D-D'-1)(D-D'-2)z=\sin(2x+3y)$$

where $D\equiv\dfrac{\partial}{\partial x},\ D'\equiv\dfrac{\partial}{\partial y}$.

(vi) Show that the equation

$$\frac{\partial^2 z}{\partial x^2}+2x\frac{\partial^2 z}{\partial x\,\partial y}+\left(1-y^2\right)\frac{\partial^2 z}{\partial y^2}=0$$

is elliptic for all values of x and y in the region $x^2+y^2<1$, parabolic on the boundary and hyperbolic out side the region.

5 Attempt any **two** of the following : **10×2=20**

(i) A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially in a position given by $y(x,0)=y_0\sin(\pi x/l)$. If it is released from rest from this position, find the displacement y at any distance x from one end at any time t.

(ii) Solve $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}=0$, $0<x<\pi$, $0<y<\pi$, which satisfies the conditions :

$u(0,y)=u(\pi,y)=u(x,\pi)=0$ and

$u(x,0)=\sin^2 x$.

(iii) A rod of length L has its ends A and B kept at $0^\circ C$ and $100^\circ C$, respectively, until steady state conditions prevail. If the temperature of B is then reduced suddenly to $0^\circ C$ and kept so, while that of A is maintained, find the temperature $u(x,t)$ at distance x from A at time t.

PAPER ID : 9917

Paper ID and Roll No. to be filled in your Answer Book

Roll No. | | | | | | | | | | |

B. Tech.

(SEM. II) EXAMINATION, 2011

MATHEMATICS - II

Time : **3** Hours] [Total Marks : **100**

Attempt all questions.

1 Attempt any **four** of following : **5×4=20**

(i) Solve : $x\,dx+y\,dy=\dfrac{a^2(x\,dy-y\,dx)}{x^2+y^2}$.

(ii) Solve : $\dfrac{dz}{dx}+\dfrac{z}{x}\log z=\dfrac{z}{x^2}(\log z)^2$.

(iii) Solve : $\left(D^2+a^2\right)y=\sec ax$.

(iv) Solve the following simultaneous equations

$$\frac{dx}{dt}+\frac{dy}{dt}-2y=2\cos t-7\sin t$$

$$\frac{dx}{dt}-\frac{dy}{dt}+2x=4\cos t-3\sin t$$

(v) Solve : $\dfrac{d^2y}{dx^2}+\dfrac{2}{x}\dfrac{dy}{dx}+\dfrac{a^2}{x^4}y=0$.

(vi) Apply the method of variation of parameters to solve $\dfrac{d^2y}{dx^2}+y=\tan x$.

2 Attempt any **two** of the following : **10×2=20**

(i) (a) If $f(t)$ is continuous and of exponential order a as $t \to \infty$, then prove that Laplace transform of $f(t)$ exists for $s > a$.

(b) Find the Laplace transform of the following periodic function :

$$f(t) = \frac{t}{T} \text{ for } 0 < t < T.$$

(ii) If $\bar{f}_1(s)$ and $\bar{f}_2(s)$ are the Laplace transforms of $f_1(t)$ and $f_2(t)$ respectively, then prove that

$$\bar{f}_1(s) \cdot \bar{f}_2(s) = L\left\{\int_0^t f_1(u) \cdot f_2(t-u)\,du\right\}.$$

Hence evaluate $L^{-1}\left\{\dfrac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$.

(iii) Solve the following simultaneous differential equations by using Laplace transformation :

$$Dx - y = e^t,\ Dy + x = \sin t,\ x_0 = 1,\ y_0 = 0.$$

3 Attempt any **two** of the following : **10×2=20**

(i) Test the convergence or divergence of the following series :

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}\ (p > 0)$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{(n+1)}}$

(ii) State and prove the Cauchy's root test. Hence test for convergence the following series

$$\sum_{n=1}^{\infty} \left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3/2}}.$$

(iii) (a) Show that the following series converges uniformly in any interval :

$$\sum_{n=1}^{\infty} \frac{\cos nx}{n^p}$$

(b) Prove that

$$\int_0^1 \left(\sum_{n=1}^{\infty} \frac{x^n}{n^2}\right) dx = \sum \frac{1}{n^2(n+1)}.$$

4 Attempt any **four** of the following : **5×4=20**

(i) Find the Fourier series for the function $f(x) = x + x^2$ in the interval $-\pi < x < \pi$. Hence show that $\dfrac{\pi^2}{6} = 1 + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \ldots\ldots\ldots.$

(ii) Obtain Fourier series for the function $f(x)$ given by $f(x) = 1 + 2x/\pi,\ -\pi \le x \le 0$

$= 1 - 2x/\pi,\ 0 \le x \le \pi$

(iii) Express $f(x) = x$ as a half-range sine series in $0 < x < 2$.

(iv) Eliminate the arbitrary function f and g from the following equation :

$v = f(x+iy) + g(x-iy)$, where $i = \sqrt{(-1)}$