4 Attempt any **two** of the following : **10×2=20**

(a) Evaluate $\iint (x+y)^2 dx\ dy$ over area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

(b) Change the order of integration and hence evaluate

$$\int_0^{2a}\int_{\frac{x^2}{4a}}^{3a-x}\left(x^2+y^2\right)dydx$$

(c) Prove that $\int_0^{\frac{\pi}{2}} \sin^p\theta\cos^q\theta d\theta = \frac{\left(\frac{p+1}{2}\right)\left(\frac{q+1}{2}\right)}{2\left(\frac{p+q+2}{2}\right)}$

5 Attempt any four of the following : **10×2=20**

(a) Find the directional derivative of the function $\phi = x^2 - y^2 + 2z^2$ at the point p(1, 2, 3) in the direction of the line PQ where Q is the point (5, 0,4).

(b) A fluid motion is given by

$$\bar{v}=(y\sin z-\sin x)i+(x\sin z+2yz)j+(xy\cos z+y^2)k$$

is the motion irrotational ? If so, find the velocity potential.

(c) Verify the Green's Theorem to evaluate the line integral $\int_c (2y^2dx + 3xdy)$, where c is the boundary of the region bounded by $y = x$ and $y = x^2$.

PAPER ID : 9916

TMA-101 **Printed Pages : 4**

Paper ID and Roll No. to be filled in your Answer Book

Roll No. | | | | | | | | | | |

B. Tech.

(SEM. III) EXAMINATION, 2011

MATHEMATICS - I (BACK PAPER)

Time : 3 Hours] [Total Marks : **100**

Note : Attempt all questions.

1 Attempt any **four** of the following : **5×4=20**

(a) Transform $\begin{bmatrix} 1 & 3 & 3 \\ 2 & 4 & 10 \\ 3 & 8 & 4 \end{bmatrix}$ into a unit matrix.

(b) Reduce the matrix A to diagonal form

$$A=\begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

(c) Find for what value of λ and μ the system of linear equations :

$$x+y+z=6$$
$$x+2y+5z=10$$
$$2x+3y+\lambda z=\mu$$

has no solution and also find the solution when $\lambda = 2$ and $\mu = 10$.

(d) Find the Eigen values of the matrix

$$\begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix}$$

(e) Find the characteristic equation of the following symmetric matrix and verify it :

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & -2 \end{bmatrix}$$

(f) Prove that the characteristic roots of a Hermitian matrix are all real.

2 Attempt any **four** of the following : **5×4=20**

(a) If $y = \log\left[x + \left(\sqrt{1+x^2}\right)\right]$, prove that

$$\left(1 + x^2\right) y_{n+2} + (2n+1) x y_{n+1} + n^2 y_n = 0$$

(b) Find the nth derivative of $\dfrac{x^2}{(x+2)(2x+3)}$.

(c) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 = -\frac{9}{(x+y+z)^2}$$

(d) State Euler's Theorem of differential calculus. Hence verify the theorem for the function

$$u = \log \frac{x^2 + y^2}{xy}.$$

(e) Obtain Taylor's expansion of $\tan^{-1} \dfrac{y}{x}$ about $(1,1)$ upto and including the second degree terms. Hence compute $f(1.1, 0.9)$.

(f) If $y_1 = \dfrac{x_2 y_3}{x_1}, y_2 = \dfrac{x_3 y_1}{x_2}, y_3 \dfrac{x_1 y_2}{x_3}$, show that the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 is 4.

3 Attempt any **two** of the following : **10×2=20**

(a) Show that the function $f(x, y) = x^3 + y^3 - 63(x+y) + 12xy$ is maximum at $(-7,-7)$ and minimum at $(3,3)$.

(b) A rectangular box, which is open at the top, has a capacity of 256 cubic feet. Determine the dimension of the box such that the least material is required for the construction of the box. Use Legrange's method of multipliers to obtain the solution.

(c) Find the possible percentage error in computing the parallel resistance r of three resistances r_1, r_2, r_3 from the formula $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$, if r_1, r_2, r_3 are each in error by plus 1.2%.