(2) Classify the partial differen)l equation 3

$$\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}+\frac{\partial^2 z}{\partial x \partial y}+\frac{\partial z}{\partial x}+z=x^2+y^2.$$

5 Attempt any two : 10×2

(a) A taut string of length 2l is fastened at both ends. The mid point of the string is taken to a height b and then released from the rest in that position. Find the displacement of the string.

(b) Solve $\frac{\partial \theta}{\partial t}=\frac{\partial^2 \theta}{\partial x^2}$ for $t>0$ and $0<x<1$; θ begin the temperature. The initial and boundary conditions are $\theta(0,t)=0, t>0$;

$\frac{\partial \theta(x,t)}{\partial t}=0$ at $x=1, t>0$;

$\theta(x,0)=x$ for $0<x<1$.

(c) A rectangular plate with insulated surfaces is l cm. wide and so long compared to its width that it may be considered infinite in the length without introducing an appreciable error. If the temperature along one short edge $y=0$ is given by $u(x,0)=100 \sin \frac{\pi x}{l}, 0<x<l$, while the two long edges $x=0$ and $x=l$ as well as the other short edges, are kept at zero temperature. Show that the steady state temperature $u(x,y)=100\sin\frac{\pi x}{l}e^{-\frac{\pi x}{l}}$.

PAPER ID : 9917

Paper ID and Roll No. to be filled in your Answer Book

Roll No. | | | | | | | | | | | |

B. Tech.

(SEM. II) (EVEN SEM.) EXAMINATION, 2013

MATHEMATICS - II

Time : **3** Hours] [Total Marks : **100**

Note : Attempt all questions, the marks assigned to each question is indicated at question itself.

1 Attempt any four : 5×4

(a) Solve $(1+e^{\frac{x}{y}})\,dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$

(b) Is the differential equation $xy\,dx+\left(2x^2+3y^2-20\right)dy=0$ exact ? If not find a suitable integrating factor and solve it.

(c) Solve $4\frac{d^2y}{dx^2}+36y=\operatorname{cosec} 3x$ by method of variation of parameters.

(d) Find the solution of $\left(D^2+4\right)y=\sinh(2x)+\pi$.

(e) Solve $\frac{dx}{dt}+y=\sin t$; $\frac{dy}{dt}+x=\cos t$.

(f) Solve the differential equation $x^2y''-xy'+y=\log(x)+\pi$.

2 Attempt any four : **5×4**

(a) Find the Laplace transform of

$$f(t)=\frac{\cos 2t-\cos 3t}{t}.$$

(b) Evaluate $L^{-1}\left\{\frac{3s^2+16s+26}{s\left(s^2+4s+13\right)}\right\}$.

(c) Find the Laplace transform of

$$f(t)=\begin{cases} t, & 0<t<a \\ 2a-t, & a<t<2a \end{cases} \text{ with } f(t+2a)=f(t).$$

(d) Sate the convolution theorem of Laplace transform and using convolution theorem -

evaluate $\int_0^{\infty}\frac{e^{-t}-e^{-3t}}{t}\,dt$.

(e) Solve, by Laplace transform,

$y''+y=\sqrt{2}\sin\sqrt{2}\,t$, $y(0)=10$, $y'(0)=0$.

(f) Solve $\frac{dx}{dt}-y=e^t;\ \frac{dy}{dt}+x=\sin t$ given that

$x(0)=1, y(0)=0$ by Laplace transform.

3 Attempt any two : **10×2**

(a) Test the convergence of the series

$$1+\frac{3}{7}x+\frac{3.6}{7.10}x^2+\frac{3.6.9}{7.10.13}x^3+\ldots$$

(b) Test the convergence and absolute convergence of the series

$$\sum_1^{\infty}(-1)^{n+1}\left[\sqrt{n^2+1}-n\right].$$

(c) (1) State Weierstress's M –test for uniform convergence. Using M-test check the uniform convergence of the series

$$\frac{1}{(1+x)^3}+\frac{2}{(2+x)^3}+\frac{3}{(3+x)^3}+\ldots,\ x\geq 0$$

(2) Prove that the sequence $\{x_n\}$ defined by $x_1=\sqrt{2}$; $x_{n+1}=\sqrt{2+x_n}$ converges to the positive root of the equation $x^2-x-2=0$.

4 Attempt any two : **10×2**

(a) Find the Fourier series of periodicity 2π for $f(x)=\begin{cases} x, & (0,\pi) \\ 2\pi-x, & (\pi,2\pi) \end{cases}$ and hence

deduce $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots=\frac{\pi^2}{8}$

(b) (1) Find the half range sine series for $f(x)=\left(x-x^2\right)$ in $0<x<\pi$ **7**

(2) Find the differential equation of all spheres whose centers lie on the z-axis. **3**

(c) (1) Expand $f(x)=x\sin x$ as a cosine series in $0<x<\pi$ and show that **7**

$$1+\frac{2}{1.3}-\frac{2}{3.5}+\frac{2}{5.7}+\ldots=\frac{\pi}{2}$$