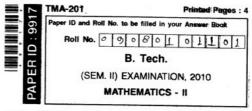
- 5 Attempt any four of the following :
- 5×4=20
- (a) Show that the G.P. $\sum_{n=0}^{\infty} r^n$, (r > 0) is convergent when r < 1 and divergent when $r \ge 1$.
- (b) Test the convergence of $\sum_{n=1}^{\infty} \cos(1/n)$
- (c) Test whether the following series is convergent or divergent $\sum \frac{1}{n} \sin(\frac{1}{n})$
- (d) By D'Alembert's ratio test, discuss the convergence of the series, $\frac{1^2 2^2}{1!} + \frac{2^2 3^2}{2!} + \frac{3^2 4^2}{3!} + \dots$
- (e) Discuss the convergence of the series,

 \frac{1}{2} + \frac{1 \cdot 3}{2 \cdot 4} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} + \ldots \ldots

 (f) By Cauchy's root test, discuss the convergence
- (f) By Cauchy's root test, discuss the convergent of the series, $\sum \left(\frac{n}{n+1}\right)^{n^2}$



Time: 3 Hours]

[Total Marks: 100

Note: Attempt all questions. All questions carry equal marks.

- 1 Attempt any four of the following: 5×4=20
 - (a) Solve $(D^4-1)y=e^x\cos x$
 - (b) Solve $x^2d^2y/dx^2 x dy/dx + y = \log x$
 - (c) Find the value of λ for which the differential equation :

$$(xy^2 + \lambda x^3y^2)dx + (x^3y + yx)xdy = 0$$
, is

exact. Hence solve.

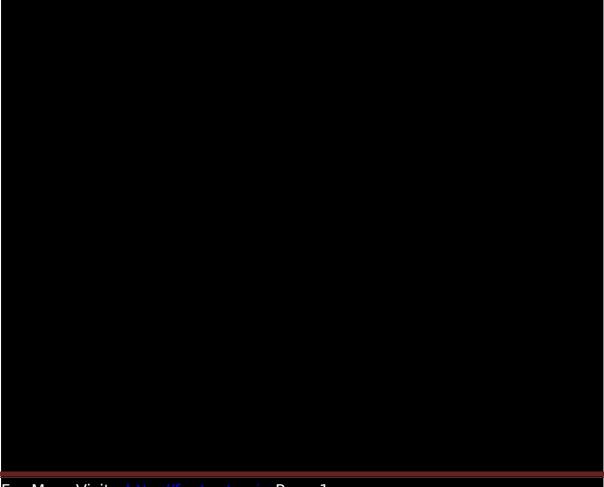
- (d) Solve by the method of variation of parameter $\frac{d^2y}{dx^2} + y = \tan x$
- (e) Solve, $(D^2 2D + 1)y = x \sin x$

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[Contd...



Solve the following simultaneous equations

$$\frac{d^2x}{dt^2} - 3x - 4y = 0, \quad \frac{d^2y}{dt^2} + x + y = 0$$

Attempt any four of the following:

(a) Find Laplace transform of

$$f(t) = \frac{\cos at - \cos bt}{t}, t > 0$$

- (b) Find $L^{-1} \left\{ \log \left(\frac{S+1}{S-1} \right) \right\}$
- (c) Find $L\left\{t^2e^{2t}\sin t\right\}$
- (d) Find by using convolution theorem

$$L^{-1} \left[\frac{S^2}{\left(S^2 + a^2\right) \left(S^2 + b^2\right)} \right]$$

$$\frac{d^2y}{dt^2} + 9y = \cos 2t, \ y(0) = 1, \ y(\pi/2) = -1$$

- (f) Evaluate by Laplace transform $\int_{0}^{\infty} t e^{-3t} \sin t dt$
- Attempt any two of the following:

(a) Find the Fourier series of the function f(x), defined as $f(x) = x \sin x$, $-\pi < x < \pi$ Hence show that

$$\frac{\pi}{2} = 1 + \frac{2}{1 \cdot 3} - \frac{2}{3 \cdot 5} + \dots$$

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[Contd...

Find the Haff-range cosine series of the following

$$f(x) = |\cos x|, \quad 0 < x < \pi$$

Draw its diagram also.

Solve the partial differential equation

$$\left(D^2 + DD' - 6D'^2\right)z = y\cos x$$

Attempt any two of the following :

(a) A tightly stretched string with fixed end points x = 0 and $x = \ell$ is initially in a position given

by
$$y(x, 0) = A \sin^3 \frac{\pi x}{\ell}$$
. It is released from rest

from this position. Find the displacement y(x, t).

(b) Solve by the method of separation of variable

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$$
; given $y(0, y) = 4e^{-y} - e^{-5y}$

(c) The temperature distribution in a bar of length π , which is perfectly insulated at ends x = 0and $x = \pi$ is governed by the partial differential

equation
$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

Assuming the initial temperature distribution as $u(x, 0) = f(x) = \cos 2x$. Find the temperature distribution at any instant of time.

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[Contd...