5 Attempt any four of the following : 5×4=20

(a) Show that the G.P. $\sum_{n=0}^{\infty} r^n, (r>0)$ is convergent when $r<1$ and divergent when $r \geq 1$.

(b) Test the convergence of $\sum_{n=1}^{\infty} \cos(1/n)$.

(c) Test whether the following series is convergent or divergent $\sum \frac{1}{n}\sin\left(\frac{1}{n}\right)$

(d) By D'Alembert's ratio test, discuss the convergence of the series,

$$\frac{1^2\,2^2}{1!}+\frac{2^2\,3^2}{2!}+\frac{3^2\,4^2}{3!}+\ldots\ldots\ldots$$

(e) Discuss the convergence of the series,

$$\frac{1}{2}+\frac{1\cdot 3}{2\cdot 4}+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}+\ldots\ldots\ldots$$

(f) By Cauchy's root test, discuss the convergence of the series, $\sum\left(\frac{n}{n+1}\right)^{n^2}$

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B. Tech.

(SEM. II) EXAMINATION, 2010

MATHEMATICS - II

Time : 3 Hours] [Total Marks : 100

Note : Attempt all questions. All questions carry equal marks.

1 Attempt any **four** of the following : 5×4=20

(a) Solve $(D^4-1)y=e^x\cos x$.

(b) Solve $x^2d^2y/dx^2-x\,dy/dx+y=\log x$

(c) Find the value of λ for which the differential equation :

$$\left(xy^2+\lambda x^3y^2\right)dx+\left(x^3y+yx\right)xdy=0,$$ is

exact. Hence solve.

(d) Solve by the method of variation of parameter

$$\frac{d^2y}{dx^2}+y=\tan x.$$

(e) Solve , $(D^2-2D+1)y=x\sin x$

(f) Solve the following simultaneous equations

$$\frac{d^2x}{dt^2} - 3x - 4y = 0, \quad \frac{d^2y}{dt^2} + x + y = 0$$

2 Attempt any four of the following : **5×4=20**

(a) Find Laplace transform of

$$f(t) = \frac{\cos at - \cos bt}{t}, t > 0$$

(b) Find $L^{-1}\left\{\log\left(\frac{S+1}{S-1}\right)\right\}$

(c) Find $L\left\{t^2 e^{2t} \sin t\right\}$

(d) Find by using convolution theorem

$$L^{-1}\left[\frac{S^2}{\left(S^2+a^2\right)\left(S^2+b^2\right)}\right]$$

(e) Solve by using Laplace transform

$$\frac{d^2y}{dt^2} + 9y = \cos 2t, \ y(0) = 1, \ y(\pi/2) = -1$$

(f) Evaluate by Laplace transform $\int_0^\infty t\, e^{-3t} \sin t\, dt$

3 Attempt any two of the following : **10×2=20**

(a) Find the Fourier series of the function $f(x)$, defined as $f(x) = x \sin x, \ -\pi < x < \pi$

Hence show that

$$\frac{\pi}{2} = 1 + \frac{2}{1\cdot 3} - \frac{2}{3\cdot 5} + \ldots\ldots$$

(b) Find the Haff-range cosine series of the following function

$$f(x) = |\cos x|, \ 0 < x < \pi$$

Draw its diagram also.

(c) Solve the partial differential equation

$$\left(D^2 + DD' - 6D'^2\right)z = y\cos x$$

4 Attempt any two of the following : **10×2=20**

(a) A tightly stretched string with fixed end points $x = 0$ and $x = \ell$ is initially in a position given by $y(x, 0) = A\sin^3 \frac{\pi x}{\ell}$. It is released from rest from this position. Find the displacement $y(x, t)$.

(b) Solve by the method of separation of variable

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u; \ \text{given} \ y(0, y) = 4e^{-y} - e^{-5y}$$

(c) The temperature distribution in a bar of length π, which is perfectly insulated at ends $x = 0$ and $x = \pi$ is governed by the partial differential equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$.

Assuming the initial temperature distribution as $u(x, 0) = f(x) = \cos 2x$. Find the temperature distribution at any instant of time.

