4 Attempt any **two** of the following : 20

(a) Prove that $B(m,n)=\frac{\lceil m \lceil n}{\lceil m+n}$; $m,n>0$.

(b) Show that $\iiint \frac{dx\,dy\,dz}{\sqrt{a^2-x^2-y^2-z^2}}=\frac{\pi^2 a^3}{8}$, the integral being extended for all +ve values of the variables for which the expression is real.

(c) Evaluate, by changing order of integration

$$\int_0^{2a}\int_{y=\frac{x^2}{a}}^{2a-x} xy\,dx\,dy$$

5 Attempt any **two** of the following : 20

(a) If $u=x+y+z,\ v=x^2+y^2+z^2,\ w=xy+zy+zx$ show that $\nabla u, \nabla v, \nabla w$ are coplanar.

(b) Prove that $\nabla^2 f(r)=f''(r)+\frac{2}{r}f'(r)$

(c) Verify Green's theorem

$\int_C \left(x^2y\,dx+x^2dy\right)$ where C is the boundary described counter clockwise of triangle with vertices (0, 0), (1, 0), (1, 1)

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TMA-101 **Printed Pages : 4**

Paper ID and Roll No. to be filled in your Answer Book

Roll No. 0908010135

B. Tech.

(SEM. I) (ODD SEM.) EXAMINATION, 2009-10

MATHEMATICS - I

Time : 3 Hours] [Total Marks : 100

1 Attempt any **four** of the following : 20

(a) Find the normal form for the matrix

$$A=\begin{bmatrix}1 & 2 & -1 & 4\\ 2 & 3 & 3 & 4\\ 1 & 2 & 3 & 4\\ -1 & -2 & 6 & -7\end{bmatrix}$$

(b) Examine if the following equations are consistent, if consistent, solve them and write the nature of solution

$3x+3y+2z=1,\ x+2y=4, 10y+3z=-2,$

$2x-3y-z=5$.

(c) Find the eigen values and eigen vectors of the matrix

$$A=\begin{bmatrix}3 & 1 & 4\\ 0 & 2 & 6\\ 0 & 0 & 5\end{bmatrix}$$

(d) If $A=\begin{bmatrix} 1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$, find the modal matrix P and resulting diagonal matrix D of A.

(e) Find the characteristic equation of the matrix

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

and hence evaluate A^{-1}.

(f) Prove that characteristic roots of unity matrix are of unit modulus.

2 Attempt any **four** of the following :

(a) If $y=\left(x^2-1\right)^n$, use Leibnitze's theorem to show that

$$\left(1-x^2\right)y_{n+2}-2xy_{n+1}+n(n+1)y_n=0$$

(b) If $u=\sin^{-1}\left(\dfrac{x+2y+3z}{\sqrt{x^8+y^8+z^8}}\right)$, show that

$xu_x+yu_y+zu_z=-3\tan u$.

(c) If $u=f(y-z, z-x, x-y)$, prove that $u_x+u_y+u_z=0$

(d) Expand x^y in powers of $(x-1)$ and $(y-1)$.

(e) If $u=x^2\tan^{-1}\left(y/x\right)-y^2\tan^{-1}\left(x/y\right)$ evaluate

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x \partial y}+y^2\frac{\partial^2 u}{\partial y^2}.$$

(f) If $u=x^2+y^2+z^2$; $v=x+y+z$, $w=xy+yz+zx$. Prove that Jacobians of (u, v, w) w.r.t. x, y, z, vanishes identically.

3 Attempt any **two** of the following :

(a) In estimating cost of a pile of bricks measured as 6 m × 50 m × 4 m the tape is stretched 1% beyond the standard length. If the count is 12 bricks in 1 m^3 and bricks cost Rs. 100 per 1000, find the approximate error in cost.

(b) Find the volume of the largest parallelopiped that can be inscribed in the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

(c) Find the maximum and minimum distances of the point (3, 4, 12) from the sphere $x^2+y^2+z^2=4$, using Lagrange's multipliers method.