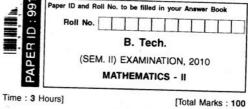
- Attempt any four of the following:
  - (a) Show that the G.P.  $\sum_{n=0}^{\infty} r^n$ , (r>0) is convergent when r < 1 and divergent when  $r \ge 1$ .
  - (b) Test the convergence of  $\sum_{n=1}^{\infty} \cos(1/n)$
  - (c) Test whether the following series is convergent or divergent  $\sum \frac{1}{n} \sin \left( \frac{1}{n} \right)$
  - (d) By D'Alembert's ratio test, discuss the convergence of the series,

$$\frac{1^2 \ 2^2}{1!} + \frac{2^2 \ 3^2}{2!} + \frac{3^2 \ 4^2}{3!} + \dots$$

(e) Discuss the convergence of the series,

$$\frac{1}{2} + \frac{1 \cdot 3}{2 \cdot 4} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} + \dots$$

(f) By Cauchy's root test, discuss the convergence of the series,  $\sum \left(\frac{n}{n+1}\right)^{n}$ 



Attempt all questions. All questions carry equal Note:

Attempt any four of the following :

5×4=20

- (a) Solve  $(D^4 1)y = e^x \cos x$ 
  - (b) Solve  $x^2 d^2 y / dx^2 x \, dy / dx + y = \log x$
  - (c) Find the value of  $\lambda$  for which the differential

$$(xy^2 + \lambda x^3y^2)dx + (x^3y + yx)xdy = 0$$
, is

exact. Hence solve.

(d) Solve by the method of variation of parameter

$$\frac{d^2y}{dx^2} + y = \tan x$$

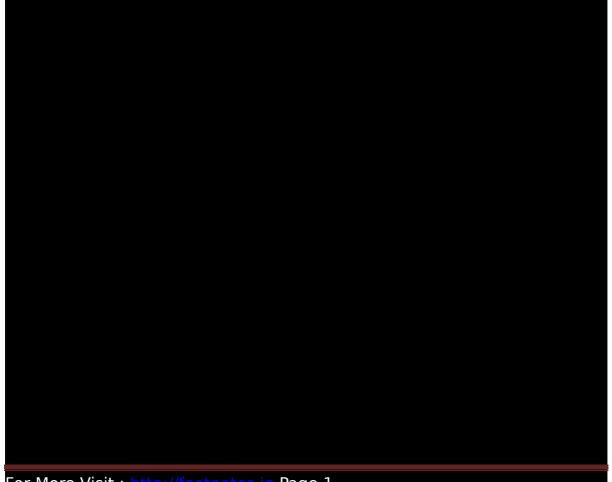
(e) Solve,  $(D^2 - 2D + 1)y = x \sin x$ 

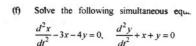
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2 Attempt any four of the following :

5×4=26

(a) Find Laplace transform of

$$f(t) = \frac{\cos at - \cos bt}{t}, t > 0$$

- (b), Find  $L^{-1}\left\{\log\left(\frac{S+1}{S-1}\right)\right\}$
- $_{\tau}$ (c) Find  $L\left\{t^2e^{2t}\sin t\right\}$ . ~
- (d) Find by using convolution theorem

$$L^{-1}\left[\frac{S^2}{\left(S^2+a^2\right)\left(S^2+b^2\right)}\right]$$

(e) Solve by using Laplace transform

$$\frac{d^2y}{dt^2} + 9y = \cos 2t, \ y(0) = 1, \ y(\pi/2) = -1$$

- (f) Evaluate by Laplace transform  $\int_0^\infty t e^{-3t} \sin t dt$
- 3 Attempt any two of the following :

10×2=2

(a) Find the Fourier series of the function f(x), defined as  $f(x) = x \sin x$ ,  $-\pi < x < \pi$ 

Hence show that  $f(x) = x \sin x, \quad -\pi < x < \pi$ 

$$\frac{\pi}{2} = 1 + \frac{2}{1 \cdot 3} - \frac{2}{3 \cdot 5} + \dots$$

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[Contd...

Find the Haff-range cosine series of the following function

$$f(x) = |\cos x|, \quad 0 < x < \pi$$

Draw its diagram also.

(c) Solve the partial differential equation

$$\left(D^2 + DD' - 6D'^2\right)z = y\cos x$$

Attempt any two of the following :

10×2=20

- (a) A tightly stretched string with fixed end points x = 0 and  $x = \ell$  is initially in a position given
  - by  $y(x, 0) = A \sin^3 \frac{\pi x}{f}$ . It is released from rest

from this position. Find the displacement y(x, t)

(b) Solve by the method of separation of variable

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$$
; given  $y(0, y) = 4e^{-y} - e^{-5y}$ 

(c) The temperature distribution in a bar of length  $\pi$ , which is perfectly insulated at ends x=0 and  $x=\pi$  is governed by the partial differential

equation 
$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

Assuming the initial temperature distribution as  $u(x, 0) = f(x) = \cos 2x$ . Find the temperature distribution at any instant of time.

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